THERMAL CONDITIONS OF ELECTRONIC INSTRUMENTS OF CASSETTE CONSTRUCTION WITH FORCED VENTILATION

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A general approach is considered for the calculation of the temperature field of a ventilated electronic instrument of cassette construction through the reduction of its heated zone to a uniform body. An approximate solution of the problem is proposed and the results of calculations are compared with experimental data.

Ventilated electronic instruments of cassette construction are widely used in technology. The search for their optimum construction is connected with the analysis of the thermal conditions of the instrument, which necessitates the improvement of the methods of calculating their temperature fields. The thermal model of an instrument of cassette construction can be represented in the form of a uniform parallelepiped with distributions of heat sources and sinks. The role of the latter is filled by gas streams passing through and carrying with them part of the energy from the instrument. A basis for this model as well as its mathematical description in the form of a system of equations for the temperature fields of the cassettes and the interlayers of gas between them are presented in [1] for the case of natural ventilation. The same system of equations will clearly also be valid for the forced ventilation of an instrument. The difference will consist only in the means of determining the flow rate of gas through the instrument: with natural ventilation additional equations are needed to calculate the gas flow rate while for forced ventilation the flow rate is assumed to be known.

The temperature field is described by the system of equations [1]

$$\lambda_1 \frac{\partial^2 \vartheta}{\partial x^2} + \lambda_2 \frac{\partial^2 \vartheta}{\partial y^2} = \alpha_v (\vartheta - \vartheta_a) - q_v, \tag{1a}$$

$$\vartheta = \vartheta_a + \frac{\omega}{\alpha_v} \frac{d\vartheta_a}{dx}, \quad \omega = \frac{c_p G}{b(h + 2\delta)} \tag{1b}$$

and the boundary conditions:

$$\left[\frac{\partial \vartheta}{\partial x} - \frac{\alpha_{x1}}{\lambda_1} \vartheta\right]_{x=0} = 0, \quad \left[\frac{\partial \vartheta}{\partial x} + \frac{\alpha_{x2}}{\lambda_1} \vartheta\right]_{x=l} = 0, \tag{2}$$

$$\left[\frac{\partial \vartheta}{\partial y} - \frac{\alpha_{y1}}{\lambda_2}\right]_{y=-a} = 0, \quad \left[\frac{\partial \vartheta}{\partial y} + \frac{\alpha_{y2}}{\lambda_2}\right]_{y=a} = 0, \tag{3}$$

$$\vartheta_a|_{x=0} = \vartheta_{in}. \tag{4}$$

In Eqs. (1) the volumetric coefficient of convective heat exchange α_v is connected with the local coefficient of heat exchange α by the dependence [1]

$$\alpha_v = \frac{2\alpha}{h + 2\delta}.$$

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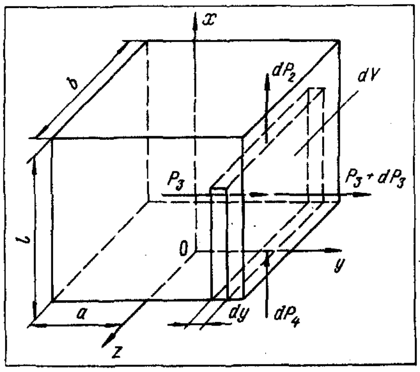

Fig. 1. For calculation of the temperature field of an electronic instrument of cassette construction.

The coordinate system is set up as shown in Fig. 1. It is assumed that the temperature variation along the width of the cassettes (along the z axis) is insignificant. The latter assumption is not necessary in principle and is adopted mainly to reduce the calculations. Moreover, in real instruments with forced ventilation the important temperature variation occurs in the direction of air movement (along the x axis) and perpendicular to the plane of the cassettes (along the y axis).

The values α_x and α_y in (2) and (3) allow for the heat exchange with the surrounding medium from the ends of the cassettes and from the faces of the heated zone which are perpendicular to the y axis, respectively. In general the conditions (2) and (3) can reflect the presence of conductive heat sinks at the boundaries of the heated zone of the instrument.

We will seek an approximate solution of the problem by the method of successive averaging of the unknown function [2, 3]. Let us introduce the averaging operator I_x

$$I_x[f] = \frac{1}{l}\int_0^l f dx$$

and designate

$$I_x[\vartheta] = \bar{\vartheta}, \quad I_x[\vartheta_a] = \bar{\vartheta}_a, \quad I_x[q_v] = \bar{q}_v,$$
$$\vartheta_a|_{x=l} = \vartheta_{out}, \quad \vartheta|_{x=l} = \vartheta_l, \quad \vartheta|_{x=0} = \vartheta_0.$$

We apply the operator I_x to all the terms of Eqs. (1), allowing for the conditions (2) and (4), and then combine the results of the termwise action of the operator

$$\lambda_2 \frac{d^2\bar{\vartheta}}{dy^2} = \frac{\alpha_{x1}}{l}\vartheta_0 + \frac{\alpha_{x2}}{l}\vartheta_l + \bar{\alpha}_v(\bar{\vartheta} - \bar{\vartheta}_a) - \bar{q}_v,$$
$$\bar{\vartheta} = \bar{\vartheta}_a + \frac{\omega}{\bar{\alpha}_v l}(\vartheta_{out} - \vartheta_{in}). \tag{5}$$

Here $\bar{\alpha}_v$ is the averaged value of the volumetric coefficient of convective heat exchange, for the calculation of which one must know the coefficients of convective heat exchange $\bar{\alpha}$ in the channels divided by the mean integral temperature difference:

$$\bar{\alpha}_v = \bar{\alpha}_v(y) = \frac{2\bar{\alpha}}{h+2\delta}, \quad \bar{\alpha} = \frac{q}{\frac{1}{l}\int_0^l (\vartheta - \vartheta_a)dx}. \tag{6}$$

Let us adopt the specific assumptions for the method of averaging [2, 3]

$$\frac{\vartheta_{out} - \vartheta_{in}}{\bar{\vartheta}_a} = \psi \neq f(y), \quad \frac{\vartheta_0}{\bar{\vartheta}} = \psi_1 \neq f_1(y), \quad \frac{\vartheta_l}{\bar{\vartheta}} = \psi_2 \neq f_2(y). \tag{7}$$

Then (5) can be rewritten in the following form:

$$\lambda_2 \frac{d^2\bar{\vartheta}}{dy^2} - \frac{\psi_1\alpha_{x1} + \psi_2\alpha_{x2}}{l}\bar{\vartheta} = \bar{\alpha}_v(\bar{\vartheta} - \bar{\vartheta}_a) - \bar{q}_v, \tag{8a}$$

$$\bar{\vartheta} = \bar{\vartheta}_a + \frac{\psi\omega}{\bar{\alpha}_v l}\bar{\vartheta}_a. \tag{8b}$$

We find the value of $\bar{\vartheta}_a$ from (8b) and substitute it into (8a):

$$\frac{d^2\bar{\vartheta}}{dy^2} - p^2\bar{\vartheta} = -\frac{\bar{q}_v}{\lambda_2}, \quad p^2 = \frac{\bar{\alpha}_v}{\lambda_2}\frac{\psi\omega}{\psi\omega + \bar{\alpha}_v l} + \frac{\psi_1\alpha_{x1} + \psi_2\alpha_{x2}}{l\lambda_2}. \tag{9}$$

Let us apply the operator I_x to the boundary conditions (3)

$$\left[\frac{d\bar{\vartheta}}{dy}-\frac{\alpha_{y1}}{\lambda_2}\bar{\vartheta}\right]_{y=-a}=0,\quad \left[\frac{d\bar{\vartheta}}{dy}+\frac{\alpha_{y2}}{\lambda_2}\bar{\vartheta}\right]_{y=a}=0. \tag{10}$$

For the solution of Eq. (9) with the condition (10) it is necessary to know the form of the dependences $\bar{\alpha}_V = F_1(y)$, $\bar{q}_V = F_2(y)$, and $\omega = F_3(y)$. We will confine ourselves to an analysis of the relatively simple case when the values of $\bar{\alpha}_V$, $\bar{q}_V$, and ω do not depend on the coordinates x and y. In addition, we assume that the air temperature at the entrance to the apparatus is equal to the temperature of the surrounding medium ($\vartheta_{in} = 0$), that heat exchange of the end surfaces of the cassettes can be neglected ($\alpha_{x1} = \alpha_{x2} = 0$), and we will examine a variant of the problem which is symmetrical relative to the x0z plane ($\alpha_{y1} = \alpha_{y2} = \alpha_y$). The latter permits us to write the symmetry condition

$$\left[\frac{\partial\bar{\vartheta}}{\partial y}\right]_{y=0}=0. \tag{11}$$

Then we obtain

$$\bar{\vartheta}=\frac{\varphi q_v}{p^2\lambda_2},\quad \varphi=1-\left(1+\frac{\lambda_2 p}{\alpha_y}\,\mathrm{th}\,ap\right)^{-1}\frac{\mathrm{ch}\,yp}{\mathrm{ch}\,ap}. \tag{12}$$

Let us return to Eq. (1a) where in accordance with [2, 3] we make the approximate substitution

$$\lambda_2\frac{\partial^2\vartheta}{\partial y^2}\approx\lambda_2\frac{d^2\bar{\vartheta}}{dy^2}. \tag{13}$$

It follows from (9) and (12) that

$$\frac{d^2\bar{\vartheta}}{dy^2}=p^2\bar{\vartheta}-\frac{q_v}{\lambda_2}=\frac{q_v}{\lambda_2}(\varphi-1). \tag{14}$$

We substitute the value $d^2\bar{\vartheta}/dy^2$ from (14) into (1a) in place of $\partial^2\vartheta/\partial y^2$ and obtain an equation which in contrast to (1a) will be approximate. In order to emphasize this let us replace the values ϑ and ϑ_a with the new designations u and u_a for the unknown approximate superheats, retaining the former meaning of the indices:

$$\lambda_1\frac{d^2u}{dx^2}=\bar{\alpha}_v(u-u_a)-\varphi q_v. \tag{15}$$

The latter equation contains the two unknown values u and u_a. A second equation can be found by using (1b), which in the end leads to the necessity of solving a differential equation of third order. To avoid an increase in the order of the differential equation let us adopt another variant of the analysis: let us use the results of the averaging already performed to find the additional dependence between the values u and u_a.

Let us isolate a volume dV = bxdy in the heated zone (see Fig. 1). The heat sources in this volume emit the power $dP = q_V = bxdy$. The superheat u_a of the air at the exit from the volume dV arises due to the heat flux $dP_1 = \omega b u_a dy$. The heat flux $dP_2 = -\lambda_1 b(du/dx)dy$ flows through the face bdy at a distance x from the entrance of air to the heated zone of the apparatus. The heat flux P_3 in the direction of the y axis flows through the volume dV, changing by the value $dP_3 = bxdy$. The change $d\bar{q}_3$ in the density $\bar{q}_3$ of the heat flux density $\bar{q}_3$ of the heat flux P_3 averaged over the x axis is equal to

$$d\bar{q}_3=-\lambda_2\frac{\partial^2}{\partial y^2}\left[\frac{1}{l}\int_0^l\vartheta dx\right]dy=-\lambda_2\frac{\partial^2\bar{\vartheta}}{\partial y^2}dy.$$

We can find $d\bar{q}_3$ by substituting into the right side of the latter equation the value $\partial^2\vartheta/\partial y^2$ from (13) and (14). Then dP_3 can be represented in the following form:

$$dP_3=(1-\varphi)\,bxq_v dx. \tag{16}$$

The heat flux dP_4 through the face bdy at x = 0 is equal to

$$dP_4=-\lambda_1 b\left|\frac{du}{dx}\right|_{x=0}dy=-\lambda_1 b\frac{\alpha_{x1}}{\lambda_1}u_0 dy=-b\psi_1\alpha_{x1}\bar{u}dy.$$

Here Eqs. (2) and (7) and the substitution $\vartheta \approx u$ are used. Because of the assumption that $\alpha_{x1} = 0$ we have $dP_4 = 0$.

According to the law of conservation of energy we have

$$dP = dP_1 + dP_2 + dP_3. \tag{17}$$

Substituting the values of the heat fluxes into (17), we obtain

$$u_a = \frac{\lambda_1}{\omega}\frac{du}{dx} + \frac{\varphi q_v}{\omega}x. \tag{18}$$

We then substitute the value of u_a found into (15):

$$\frac{d^2u}{dx^2} + \frac{\bar{\alpha}_v}{\omega}\frac{du}{dx} - \frac{\bar{\alpha}_v}{\lambda_1}u = -\frac{\varphi q_v}{\lambda_1} - \frac{\bar{\alpha}_v}{\lambda_1}\frac{\varphi q_v}{\omega}x. \tag{19}$$

By integrating Eq. (19), using Eq. (18), the boundary conditions, and the assumptions adopted, we obtain the following approximate equations for u_a and u:

$$\frac{\omega}{\varphi q_v}u_a = \frac{\lambda_1}{\omega}[\gamma \exp x\beta_1 - (1+\gamma)\exp x\beta_2] + x, \tag{20a}$$

$$\frac{\omega}{\varphi q_v}u = \frac{\gamma}{\beta_1}\exp x\beta_1 - \frac{1+\gamma}{\beta_2}\exp x\beta_2 + x + \frac{\lambda_1}{\omega} + \frac{\omega}{\bar{\alpha}_v}, \tag{20b}$$

where

$$\beta_{1,2} = -\frac{\bar{\alpha}_v}{2\omega}(1 \pm \sqrt{1+k}), \quad k = 4\frac{\omega^2}{\bar{\alpha}_v\lambda_1}, \quad \gamma = \frac{1-\exp\beta_2 l}{\exp\beta_2 l - \exp\beta_1 l}.$$

The solution (20) will have a simpler form if in the derivation of the equations we take $\lambda_1 \to 0$ (constant flux density from the surface of the cassettes) or $\lambda_1 \to \infty$ (the temperature of the cassettes does not vary in the direction of air movement). In the first case we will have

$$u_a = \frac{\varphi q_v}{\omega}x, \quad u = u_a + \frac{\varphi q_v}{\bar{\alpha}_v}, \tag{21}$$

and in the second case

$$u_a = u\left[1 - \exp\left(-\frac{\bar{\alpha}_v}{\omega}x\right)\right], \quad u = \frac{\varphi q_v l}{\omega}\left[1 - \exp\left(-\frac{\bar{\alpha}_v l}{\omega}\right)\right]^{-1}. \tag{22}$$

The transition to Eqs. (21) and (22) does not give a substantial difference from a calculation according to Eqs. (20) if $k > 10$ or $k < 0.01$, respectively.

The multiplier φ in (20)-(22) is given in (12) and allows for the heat exchange of the lateral surface of the heated zone of the instrument with the surrounding medium. At $\varphi = 1$ the surrounding medium does not affect the temperature of the cassettes. In order to determine φ one must first find the values of ψ_i from Eqs. (7) by substituting into them the superheats calculated using (20)-(22) at $\varphi = 1$. After this Eqs. (9) and (12) permit an estimate of the variation in the temperature field in the direction of the y axis. Having determined $\varphi = \varphi(y)$ one can approximately calculate the two-dimensional temperture field of the heated zone from Eqs. (20)-(22). Further refinement can be accomplished by the method of successive approximations.

For practical calculations of the temperature field one must first calculate the values $\bar{\alpha}$, ω, λ_1, and λ_2. The calculation of the last three values is examined in sufficient detail in [1]. Simplified dependences for the determination of $\bar{\alpha}$ are presented in [4] for cassettes with smooth surfaces.

The results of a calculation of the temperature field by the approximate method suggested are compared with the data of an experimental study of several models of typical constructions of cassette electronic instruments [5]. The calculated (u) and measured (ϑ) superheats with respect to the surrounding medium were compared at each experimental point:

$$\Delta_i = \frac{u_i - \vartheta_i}{\vartheta_i}\cdot 100\%.$$

The series of studies of the m superheats compared for each model was generalized by the dependence

$$\sigma = \frac{1}{m-1}\sum_{i=1}^{m}\Delta_i^2 . \qquad (23)$$

For each model the value of σ did not exceed 12%.

NOTATION

x	is the coordinate coinciding with direction of air movement;
y	is the coordinate perpendicular to plane of cassettes;
l, $2a$, b	are the dimensions of heated zone in the directions of the x, y, and z axes, respectively;
2δ	is the thickness of cassette;
h	is the distance between cassettes;
ϑ, u	are the superheats of surface of cassettes relative to temperature of surrounding medium;
ϑ_a, u_a	are the mean flow rate superheat of air in gap between cassettes;
α	is the local coefficient of convective heat exchange in channel;
α_V	is the volumetric coefficient of convective heat exchange;
α_x, α_y	are the coefficients of heat transfer from heated zone to surrounding medium;
λ_1, λ_2	are the effective coefficients of thermal conductivity of heated zone;
c_p, G	are the specific heat capacity and mass flow rate of air in channel between cassettes;
q_V	is the volumetric density of heat sources;
upper bar	is the averaged value of parameter.

LITERATURE CITED

1. G. N. Dul'nev and N. N. Tarnovskii, Thermal Conditions of an Electronic Instrument [in Russian], Énergiya (1971).
2. A. Akaev and G. N. Dul'nev, Inzh.-Fiz. Zh., 22, No. 4 (1972).
3. A. Akaev and G. N. Dul'nev, Transactions of Leningrad Institute of Precision Mechanics and Optics [in Russian], No. 70, Leningrad (1972).
4. Yu. A. Gavrilov and G. N. Dul'nev, Inzh.-Fiz. Zh., 23, No. 4 (1972).
5. É. F. Buts, Yu. A. Gavrilov, and N. N. Tarnovskii, Ref. 382, Ref. Inform. po Radioelektronike, No. 10 (1972).